

# HOW TO CALCULATE THE QUANTUM PART OF THE TRULY NONPERTURBATIVE YANG-MILLS VACUUM ENERGY DENSITY IN THE AXIAL GAUGE QCD

V. Gogohia and Gy. Kluge

*HAS, CRIP, RMKI, Theory Division, P.O.B. 49, H-1525 Budapest 114, Hungary*

*email address: gogohia@rmki.kfki.hu*

## Abstract

Using the effective potential approach for composite operators, we have formulated a general method how to calculate the truly nonperturbative vacuum energy density in the axial gauge QCD quantum models of its ground state. It is defined as integrated out the truly nonperturbative part of the full gluon propagator over the deep infrared region (soft momentum region). The non-trivial minimization procedure makes it possible to determine the value of the soft cutoff in terms of the corresponding nonperturbative scale parameter which is inevitably presented in any nonperturbative model for the full gluon propagator. If the chosen Ansatz for the full gluon propagator is a realistic one, then our method uniquely determines the truly vacuum energy density, which is always finite, automatically negative and it has no imaginary part (stable vacuum). We illustrate it by considering the Abelian Higgs model of dual QCD ground state. We have explicitly shown that the vacuum of this model without string contributions is unstable against quantum corrections.

PACS numbers: 11.15.Tk, 12.38.Lg

## I. INTRODUCTION

The nonperturbative QCD vacuum is a very complicated medium and its dynamical and topological complexity [1-3] means that its structure can be organized at various levels (classical, quantum) and it can contain many different components and ingredients which contribute to the vacuum energy density (VED), one of the main characteristics of the QCD ground state. Many models of the QCD vacuum involve some extra classical color field configurations such as randomly oriented domains of constant color magnetic fields, background gauge fields, averaged over spin and color, stochastic colored background fields, etc. (see Refs. [1,4] and references therein). The most elaborated classical models are random and interacting instanton liquid models (RILM and IILM, respectively) of the QCD vacuum [5]. These models are based on the existence of the topologically nontrivial instanton-type fluctuations of gluon fields, which are nonperturbative solutions to the classical equations of motion in Euclidean space (see Ref. [5] and references therein).

Here we are going to discuss the quantum part of VED which is determined by the effective potential approach for composite operators [6,7] (see also Ref. [8]). It allows us to investigate the nonperturbative QCD vacuum, in particular Yang-Mills (YM) one, by substituting some physically well-justified Ansatz for the full gluon propagator since the exact solutions are not known. In the absence of external sources the effective potential is nothing but VED which is given in the form of the loop expansion where the number of the vacuum loops (consisting in general of the confining quarks and nonperturbative gluons) is equal to the power of the Plank constant,  $\hbar$ .

Let us remind the reader that the full dynamical information of any quantum gauge field theory such as QCD is contained in the corresponding quantum equations of motion, the so-called Schwinger-Dyson (SD) equations for lower (propagators) and higher (vertices and kernels) Green's functions [9,10]. These equations should be also complemented by the corresponding Slavnov-Taylor (ST) identities [9-12] which in general relate the above mentioned lower and higher Green's functions to each other. These identities are consequences of the exact gauge invariance and therefore "*are exact constraints on any solution to QCD*" [9]. Precisely this system of equations can serve as an adequate and effective tool for the nonperturbative approach to QCD. Among the above-mentioned Green's functions, the two-point Green's function describing the full gluon propagator (see section II below) has a central place [9-13]. In particular, the solutions to the above-mentioned SD equation for the full gluon propagator, are supposed to reflect the quantum structure of the QCD ground state. It is a highly nonlinear integral equation containing many different propagators, vertices and kernels [9-13]. For this reason it may have many different exact solutions with different asymptotics in the deep infrared (IR) limit (the ultraviolet (UV) asymptotics because of asymptotic freedom are apparently uniquely determined), describing thus many different types of quantum excitations of gluon field configurations in the QCD vacuum. Evidently, there is no hope for an exact solutions as well as not all of them can reflect the real structure of the QCD vacuum. Let us emphasize now that any deviation in the behavior of the full gluon propagator in the IR domain from the free one automatically assumes its dependence on a scale parameter (at least one) responsible for nonperturbative dynamics in the quantum model under consideration, say,  $\Lambda_{NP}$ . This is very similar to asymptotic freedom which requires asymptotic scale parameter associated with the nontrivial perturbative dynamics

(scale violation). However, to calculate the truly nonperturbative VED we need not the IR part in the decomposition of the full gluon propagator, but rather its truly nonperturbative part which vanishes when the above-mentioned nonperturbative scale parameter goes to zero, i.e., when the perturbative phase survives only in the corresponding decomposition of the full gluon propagator (see next section below).

It is well known, however, that VED is badly divergent in quantum field theory, in particular QCD (see, for example, the discussion given by Shifman in Ref. [1]). The main problem thus is how to extract the truly nonperturbative VED which is relevant for the QCD vacuum quantum model under consideration. It should be finite, negative and it should have no imaginary part (stable vacuum). Why is it so important to calculate it from first principles? As was emphasized above, this quantity is important in its own right being nothing but the bag constant (the so-called bag pressure) apart from the sign, by definition [14]. Through the trace anomaly relation [15] it assists in the correct estimating such an important phenomenological nonperturbative parameter as the gluon condensate introduced in the QCD sum rules approach to resonance physics [16]. Furthermore, it assists in the resolution of the  $U(1)$  problem [17] via the Witten-Veneziano (WV) formula for the mass of  $\eta'$  meson [18]. The problem is that the topological susceptibility needed for this purpose [16-19] is determined by the two point correlation function from which perturbative contribution is already subtracted by definition [18-22]. The same is valid for the above-mentioned bag constant which is much more general quantity than the string tension since it is relevant for light quarks as well. Thus to correctly calculate the truly nonperturbative VED means to correctly understand the structure of the QCD vacuum in different models.

We have already formulate a method how to calculate the truly nonperturbative YM VED in the covariant gauge QCD [23]. The main purpose of this paper (section II) is to formulate precisely a general method how to correctly calculate the truly nonperturbative quantum part of YM VED in the axial gauge QCD. In sections III and IV we illustrate it by considering the Abelian Higgs model [24] of the dual QCD [25] ground state. We will explicitly show that the vacuum of this model without string contributions is unstable against quantum corrections. In section V we summarize our results.

## II. THE TRULY NONPERTURBATIVE VACUUM ENERGY DENSITY

In this section we are going to analytically formulate a general method of calculation of the quantum part of the truly nonperturbative YM VED in the axial gauge QCD. Let us start from the nonperturbative gluon part of VED which to-leading order (log-loop level  $\sim \hbar$ )<sup>1</sup> is given by the effective potential for composite operators [6] as follows

$$V(D) = \frac{i}{2} \int \frac{d^n q}{(2\pi)^n} \text{Tr} \{ \ln(D_0^{-1} D) - (D_0^{-1} D) + 1 \}, \quad (2.1)$$

where  $D(q)$  is the full gluon propagator (see below) and  $D_0(q)$  is its free (perturbative) counterpart. Here and below the traces over space-time and color group indices are understood.

---

<sup>1</sup>Next-to-leading and higher terms (two and more vacuum loops) are suppressed by one order of magnitude in powers of  $\hbar$  at least and are left for consideration elsewhere.

The effective potential is normalized as  $V(D_0) = 0$ , i.e., the free perturbative vacuum is normalized to zero.

A general parametrization of the gauge boson propagator in the axial gauge of dual QCD is [24-26] (here and below we use notations and definitions of Refs. [24,26])

$$D_{\mu\nu}(q, n) = -\frac{1}{(q \cdot n)^2} T_{\mu\nu}(n) G(-q^2) + L_{\mu\nu}(q, n) F(-q^2), \quad (2.2)$$

where

$$\begin{aligned} T_{\mu\nu}(n) &= \delta_{\mu\nu} - n_\mu n_\nu, \\ L_{\mu\nu}(q, n) &= \delta_{\mu\nu} - \frac{q_\mu n_\nu + q_\nu n_\mu}{(q \cdot n)} + \frac{q_\mu q_\nu}{(q \cdot n)^2} \end{aligned} \quad (2.3)$$

with an arbitrary constant unit vector  $n_\mu$ ,  $n_\mu^2 = 1$ . The exact coefficient functions  $G(-q^2)$  and  $F(-q^2)$  characterize the vacuum of the theory under consideraion. Their free perturbative counterparts are

$$F^{PT}(-q^2) = \frac{1}{(-q^2)}, \quad G^{PT}(-q^2) = 0. \quad (2.4)$$

Thus the free perturbative gluon propagator is

$$D_{\mu\nu}^0(q, n) = \frac{1}{(-q^2)} L_{\mu\nu}(q, n) \quad (2.5)$$

while its inverse is

$$[D_{\mu\nu}^0]^{-1}(q) = (-q^2) \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right). \quad (2.6)$$

Using futher Eqs. (2.2) and (2.6), one obtains

$$[D_{\mu\nu}^0]^{-1}(q) D_{\mu\nu}(q, n) = (-q^2) F(-q^2) + G(-q^2). \quad (2.7)$$

In order to evaluate the effective potential (2.1) we use the well-known expression,

$$Tr \ln(D_0^{-1} D) = 8 \times \ln \det(D_0^{-1} D) = 8 \times 4 \ln [(-q^2) F(-q^2) + G(-q^2)]. \quad (2.8)$$

It becomes zero (in accordance with the above mentioned normalization condition) when the full gluon form factors are replaced by their free counterparts (see Eqs. (2.4)). Going over to four ( $n = 4$ ) dimensional Euclidean space in Eq. (2.1), on account of (2.8), and evaluating some numerical factors, one obtains ( $\epsilon_g = V(D)$ )

$$\epsilon_g = -\frac{1}{\pi^2} \int dq^2 \, q^2 \left[ \ln (q^2 F(q^2) + G(q^2)) - (q^2 F(q^2) + G(q^2)) + 1 \right]. \quad (2.9)$$

Let us now introduce the following decomposition of the exact coefficient functions  $G(q^2)$  and  $F(q^2)$  (Euclidean metrics)

$$\begin{aligned} F(q^2) &= F^{NP}(q^2) + F^{PT}(q^2), \\ G(q^2) &= G^{NP}(q^2) + G^{PT}(q^2), \end{aligned} \quad (2.10)$$

where the truly nonperturbative quantities  $F^{NP}(q^2)$  and  $G^{NP}(q^2)$  are defined as follows:

$$\begin{aligned} F^{NP}(q^2, \Lambda_{NP}) &= F(q^2, \Lambda_{NP}) - F(q^2, \Lambda_{NP} = 0), \\ G^{NP}(q^2, \Lambda_{NP}) &= G(q^2, \Lambda_{NP}) - G(q^2, \Lambda_{NP} = 0), \end{aligned} \quad (2.11)$$

which explains the difference between the truly nonperturbative parts and the full gluon form factors which are nonperturbative themselves. Let us note, that the perturbative parts  $F^{PT}(q^2)$  and  $G^{PT}(q^2)$  may, in general, contain renormgroup log improvmenets due to asymptotic freedom. Without these improvments their free perturbative counterparts are given in Eqs. (2.4). Substituting these relations into Eq. (2.9) and doing some trivial rearangment, one obtains

$$\epsilon_g = -\frac{1}{\pi^2} \int dq^2 \, q^2 \left[ \ln \left( 1 + q^2 F^{NP}(q^2) + G^{NP}(q^2) \right) - \left( q^2 F^{NP}(q^2) + G^{NP}(q^2) \right) \right] + I_{PT}, \quad (2.12)$$

where we introduce the following notation

$$I_{PT} = -\frac{1}{\pi^2} \int dq^2 \, q^2 \left[ \ln \left( 1 - \frac{1 - q^2 F^{PT}(q^2) - G^{PT}(q^2)}{1 + q^2 F^{NP}(q^2) + G^{NP}(q^2)} \right) + \left( 1 - q^2 F^{PT}(q^2) - G^{PT}(q^2) \right) \right], \quad (2.13)$$

as containing contribution which is mainly determined by the perturbative part. However, this is not the whole story yet. We must now to introduse the soft cutoff in order to separate the deep IR region where the truly nonperturbative contributions become dominant (obviously they can not be valid in the whole energy-momentum range). So the expression (2.12) becomes

$$\epsilon_g = -\frac{1}{\pi^2} \int_0^{q_0^2} dq^2 \, q^2 \left[ \ln \left( 1 + q^2 F^{NP}(q^2) + G^{NP}(q^2) \right) - \left( q^2 F^{NP}(q^2) + G^{NP}(q^2) \right) \right] + I_{PT} + \tilde{I}_{PT}, \quad (2.14)$$

where the explicit formula for  $\tilde{I}_{PT}$  (which is obvious) is not important. The contribution over the perturbative region  $\tilde{I}_{PT}$  as well as  $I_{PT}$  should be subtracted by introducing the corresponding counter terms into the effective potential, which is equivalent to define the truly nonperturbative VED as  $\epsilon_g^{np} = \epsilon_g - I_{PT} - \tilde{I}_{PT}$ . Thus one finally obtains

$$\epsilon_g^{np} = -\frac{1}{\pi^2} \int_0^{q_0^2} dq^2 \, q^2 \left[ \ln \left( 1 + q^2 F^{NP}(q^2) + G^{NP}(q^2) \right) - \left( q^2 F^{NP}(q^2) + G^{NP}(q^2) \right) \right]. \quad (2.15)$$

This a general formula which can be applied to any model of the axial gauge QCD ground state based on the corresponding Ansatz for the full gluon propagator. So Eq. (2.15) is our definition of the truly nonperturbative VED as integrated out the truly nonperturbative part of the full gluon propagator over the deep IR region, soft momentum region,  $0 \leq q^2 \leq q_0^2$ . How to determine  $q_0^2$ ? By the corresponding minimization procedure, of course (see below).

## A.

From this point it is convenient to factorize the dependence on a scale in the nonperturbative VED (2.15). As was already emphasized above, the full gluon form factors always contain at least one scale parameter responsible for the nonperturbative dynamics in the model under consideration,  $\Lambda_{NP}$ . Within our general method we are considering it as free one, i.e., as "running" (when it formally goes to zero, only perturbative phase survives in the model under consideration) and its numerical value (if any) will be used only at final stage in order to numerically evaluate the corresponding truly nonperturbative VED (if any). We can introduce dimensionless variables and parameters by using completely extra scale (which is always fixed in comparison with  $\Lambda_{NP}$ ), for example flavorless QCD asymptotic scale parameter  $\Lambda_{YM}$  as follows:

$$z = \frac{q^2}{\Lambda_{YM}^2}, \quad z_0 = \frac{q_0^2}{\Lambda_{YM}^2}, \quad b = \frac{\Lambda_{NP}^2}{\Lambda_{YM}^2}. \quad (2.16)$$

Here  $z_0$  is the corresponding dimensionless soft cutoff while the parameter  $b$  has a very clear physical meaning. It measures the ratio between nonperturbative dynamics, symbolized by  $\Lambda_{NP}^2$  and nontrivial perturbative dynamics (violation of scale, asymptotic freedom) symbolized by  $\Lambda_{YM}^2$ . When it is zero only perturbative phase remains in the quantum model under consideration. In this case, the gluon form factors obviously become a functions of  $z$  and  $b$ , i.e.,  $F^{NP}(q^2) = F^{NP}(z, b)$  and  $G^{NP}(q^2) = G^{NP}(z, b)$ , so the truly nonperturbative VED (2.15) is ( $\epsilon_g^{np} \equiv \epsilon_g^{np}(z_0, b)$ )

$$\Omega_g(z_0, b) = \frac{1}{\Lambda_{YM}^4} \epsilon_g^{np}(z_0, b), \quad (2.17)$$

where for further aims we introduce the gluon effective potential at a fixed scale  $\Lambda_{YM}$ , [23,27]

$$\Omega_g \equiv \Omega_g(z_0, b) = \frac{1}{\pi^2} \int_0^{z_0} dz \, z \left[ \left( z F^{NP}(z, b) + G^{NP}(z, b) \right) - \ln \left( 1 + z F^{NP}(z, b) + G^{NP}(z, b) \right) \right]. \quad (2.18)$$

Precisely this expression allows us to investigate the dynamical structure of the YM vacuum free of scale dependence complications as it has been already factorized in Eq. (2.17). It depends only on  $z_0$  and  $b$  and the minimization procedure can be done now with respect to  $b$ ,  $\partial \Omega_g(z_0, b) / \partial b = 0$  (usually after integrated out in Eq. (2.18)) in order to find self-consistent relation between  $z_0$  and  $b$ , which means to find  $q_0$  as a function of  $\Lambda_{NP}$ . Let us note in advance that all final numerical results will always depend only on  $\Lambda_{NP}$  as it should be for the nonperturbative part of VED. Obviously, the minimization with respect to  $z_0$  leads to trivial zero. In principle, through the relation  $\Lambda_{YM}^4 = q_0^4 z_0^{-2}$ , it is possible to fix the soft cutoff  $q_0$  itself, but this is not the case indeed since then  $z_0$  can not be varied.

## B.

On the other hand, the scale dependence can be factorized as follows:

$$z = \frac{q^2}{\Lambda_{NP}^2}, \quad z_0 = \frac{q_0^2}{\Lambda_{NP}^2}, \quad (2.19)$$

i.e.,  $b = 1$ . For simplicity (but not loosing generality) we use the same notations for the dimensionless set of variables and parameters as in Eq. (2.16). In this case, the gluon form factors obviously becomes the function of  $z$  only and the truly nonperturbative VED (2.15) becomes

$$\epsilon_g^{np}(z_0) = \frac{1}{\pi^2} q_0^4 z_0^{-2} \int_0^{z_0} dz \, z \left[ \left( z F^{NP}(z) + G^{NP}(z) \right) - \ln \left( 1 + z F^{NP}(z) + G^{NP}(z) \right) \right]. \quad (2.20)$$

Evidently, to fix the scale now is possible in the two different ways. In principle, we can fix  $\Lambda_{NP}$  itself, i.e., introducing

$$\tilde{\Omega}_g(z_0) = \frac{1}{\Lambda_{NP}^4} \epsilon_g^{np}(z_0) = \frac{1}{\pi^2} \int_0^{z_0} dz \, z \left[ \left( z F^{NP}(z) + G^{NP}(z) \right) - \ln \left( 1 + z F^{NP}(z) + G^{NP}(z) \right) \right]. \quad (2.21)$$

However, the minimization procedure again leads to the trivial zero, which shows that this scale can not be fixed.

In contrast to the previous case, let us fix the soft cutoff itself, i.e., setting [23,28]

$$\bar{\Omega}_g(z_0) = \frac{1}{q_0^4} \epsilon_g^{np}(z_0) = \frac{1}{\pi^2} z_0^{-2} \int_0^{z_0} dz \, z \left[ \left( z F^{NP}(z) + G^{NP}(z) \right) - \ln \left( 1 + z F^{NP}(z) + G^{NP}(z) \right) \right]. \quad (2.22)$$

The minimization procedure with respect to  $z_0$  is nontrivial now. Indeed,  $\partial \bar{\Omega}_g(z_0) / \partial z_0 = 0$ , yields the following "stationary" condition

$$\begin{aligned} & \int_0^{z_0} dz \, z \left[ \left( z F^{NP}(z) + G^{NP}(z) \right) - \ln \left( 1 + z F^{NP}(z) + G^{NP}(z) \right) \right] \\ &= \frac{1}{2} z_0^2 \left[ \left( z_0 F^{NP}(z_0) + G^{NP}(z_0) \right) - \ln \left( 1 + z_0 F^{NP}(z_0) + G^{NP}(z_0) \right) \right], \end{aligned} \quad (2.23)$$

which solutions (if any) allows one to find  $q_0$  as a function of  $\Lambda_{NP}$ . On account of this "stationary" condition, the effective potential (2.22) itself becomes simpler for numerical calculations, namely

$$\bar{\Omega}_g(z_0^{st}) = \frac{1}{2\pi^2} \left[ \left( z_0^{st} F^{NP}(z_0^{st}) + G^{NP}(z_0^{st}) \right) - \ln \left( 1 + z_0^{st} F^{NP}(z_0^{st}) + G^{NP}(z_0^{st}) \right) \right], \quad (2.24)$$

where  $z_0^{st}$  is a solution (if any) of the "stationary" condition (2.23) and corresponds to the minimum(s) (if any) of the effective potential (2.22). In the next sections we will illustrate how this method works.

### III. ABELIAN HIGGS MODEL

Let us now consider some special model of the dual QCD [25] ground state. In the dual Abelian Higgs theory which confines electric charges the coefficient functions  $F(q^2)$  and  $G(q^2)$  are [24] (Euclidean metrics)

$$\begin{aligned}
F(q^2) &= \frac{1}{q^2 + M_B^2} \left( 1 + \frac{M_B^4 D^\Sigma(q^2)}{q^2 + M_B^2} \right), \\
G(q^2) &= -\frac{M_B^2}{q^2 + M_B^2} \left( 1 - M_B^2 \frac{q^2 D^\Sigma(q^2)}{q^2 + M_B^2} \right),
\end{aligned} \tag{3.1}$$

where  $M_B$  is the mass of the dual gauge boson  $B_\mu$  and  $D^\Sigma(q^2)$  represents the string contribution into the gauge boson propagator. The mass scale parameter  $M_B$  is the scale responsible for nonperturbative dynamics in this model (in our notations  $\Lambda_{NP} = M_B$ ). When it formally goes to zero, then one recovers the free perturbative expressions indeed, (2.4). Removing the string contributions from these relations we get

$$F^{no-str.}(q^2) = \frac{1}{q^2 + M_B^2}, \quad G^{no-str.}(q^2) = -\frac{M_B^2}{q^2 + M_B^2}, \tag{3.2}$$

i.e., even in this case these quantities remain nonperturbative. The truly nonperturbative expressions (2.11) now become

$$\begin{aligned}
F^{NP}(q^2) &= -\frac{M_B^2}{q^2(q^2 + M_B^2)} \left( 1 - \frac{M_B^2 q^2 D^\Sigma(q^2)}{q^2 + M_B^2} \right), \\
G^{NP}(q^2) &= -\frac{M_B^2}{q^2 + M_B^2} \left( 1 - \frac{M_B^2 q^2 D^\Sigma(q^2)}{q^2 + M_B^2} \right),
\end{aligned} \tag{3.3}$$

while with no-string contributions they are

$$F_{NP}^{no-str.}(q^2) = -\frac{M_B^2}{q^2(q^2 + M_B^2)}, \quad G_{NP}^{no-str.}(q^2) = -\frac{M_B^2}{q^2 + M_B^2}. \tag{3.4}$$

Both expressions (3.3) and (3.4) are truly nonperturbative indeed, since they become zero in the perturbative limit ( $M_B \rightarrow 0$ ), when only perturbative phase remains. From these relations also follows

$$\begin{aligned}
G^{NP}(q^2) &= q^2 F^{NP}(q^2) = -\frac{M_B^2}{q^2 + M_B^2} \left( 1 - \frac{M_B^2 q^2 D^\Sigma(q^2)}{q^2 + M_B^2} \right), \\
G_{NP}^{no-str.}(q^2) &= q^2 F_{NP}^{no-str.}(q^2) = -\frac{M_B^2}{q^2 + M_B^2},
\end{aligned} \tag{3.5}$$

so the truly nonperturbative vacuum energy density (2.15) will depend only on one function, say,  $G^{NP}(q^2)$  (see next section).

Although the expression (2.2), on account of (3.1), for the gluon propagator is exact, nevertheless it contains an unknown function  $D^\Sigma(q^2)$  which is the intermediate string state contribution into the gauge boson propagator [24]. It can be considered as a glueball state with the photon quantum numbers  $1^-$ . The behavior of this function  $D^\Sigma(q^2)$  in the IR region ( $q^2 \rightarrow 0$ ) can be estimated as follows [24]:

$$D^\Sigma(q^2) = \frac{C}{q^2 + M_{gl}^2} + \dots, \tag{3.6}$$



where  $C$  is a dimensionless parameter and  $M_{gl}^2$  is the mass of the lowest  $1^-$  glueball state. The dots denote the contributions of heavier states. Thus, according to Eqs. (3.1) and (3.6), the coefficient functions in the IR limit behave like

$$F(q^2) = \frac{1}{M_B^2} + \frac{C}{M_{gl}^2} + O(q^2), \quad G(q^2) = -1 + O(q^2), \quad q^2 \rightarrow 0. \quad (3.7)$$

At the same time according to Eqs. (3.3), (3.5) and (3.6) their truly nonperturbative counterparts behave like  $G^{NP}(q^2) = q^2 F^{NP}(q^2) = -1 + O(q^2)$ ,  $q^2 \rightarrow 0$ , i.e., in the same way as  $G(q^2)$  in Eq. (3.7).

#### IV. VACUUM STRUCTURE IN THE ABELIAN HIGGS MODEL

Let us calculate the truly nonperturbative VED in the Abelian Higgs model described in the preceding section. It is instructive to start from the case when there are no string contributions into the structure functions  $F(q^2)$  and  $G(q^2)$ . Then their truly nonperturbative parts are given in Eqs. (3.4). It is convenient to factorize the scale dependence of VED by introducing dimensionless variables and parameters in accordance with B-scheme (2.19) with  $\Lambda_{NP} = M_B$ . In this case, the gluon form factors (structure functions) obviously becomes the functions of  $z$  only,  $q^2 F^{NP}(q^2) = G^{NP}(q^2) = -(1/(1+z))$ . The truly nonperturbative VED (2.22) becomes

$$\bar{\Omega}_g(z_0) = \frac{1}{q_0^4} \epsilon_g^{np}(z_0) = -\frac{1}{\pi^2} z_0^{-2} \int_0^{z_0} dz \, z \left[ \frac{2}{1+z} + \ln \left( \frac{-1+z}{1+z} \right) \right]. \quad (4.1)$$

Easily integrating Eq. (4.1), one obtains

$$\bar{\Omega}_g(z_0) = -\frac{1}{2\pi^2} z_0^{-2} \left[ 2z_0 - 4 \ln(1+z_0) + \ln \left( \frac{1+z_0}{1-z_0} \right) + z_0^2 \ln \left( \frac{-1+z_0}{1+z_0} \right) \right]. \quad (4.2)$$

From this expression it is almost obvious that the effective potential will have imaginary part at any finite value of the soft cutoff, which is a direct manifestation of the vacuum instability [29]. Asymptotics of the effective potential (4.2) to-leading order are

$$\begin{aligned} \bar{\Omega}_g(z_0)_{z_0 \rightarrow 0} &\sim -\frac{1}{2\pi^2} \ln(-1), \\ \bar{\Omega}_g(z_0)_{z_0 \rightarrow \infty} &\sim \frac{2}{\pi^2} z_0^{-2} \ln z_0. \end{aligned} \quad (4.3)$$

Let us remind, that  $z_0 \rightarrow \infty$  is the perturbative limit ( $M_B \rightarrow 0$ ) when the soft cutoff  $q_0$  is fixed. The "stationary" condition (2.23) now is

$$4 \ln(1+z_0) - \ln \left( \frac{1+z_0}{1-z_0} \right) = \frac{z_0}{1+z_0} \left( (1+z_0)^2 + 2 \right). \quad (4.4)$$

It has only trivial solution  $z_0 = 0$ , so the "stationary" state does not exist in this model.

## V. CONCLUSIONS

In summary, we have formulated a general method how to calculate the truly nonperturbative VED in the axial gauge QCD quantum models of its ground state using the effective potential approach for composite operators. It is defined as integrated out the truly nonperturbative part of the full gluon propagator over the deep IR region (soft momentum region). The nontrivial minimization procedure which can be done only by the two different ways (leading however to the same numerical value (if any) of VED) makes it possible to determine the value of the soft cutoff in terms of the corresponding nonperturbative scale parameter which is inevitably presented in any nonperturbative model for the full gluon propagator. If the chosen Ansatz for the full gluon propagator is a realistic one, then our method uniquely determines the truly nonperturbative VED, which is always finite, automatically negative and it has no imaginary part (see, for example our previous publications [23,28]). Here we illustrate it by considering the Abelian Higgs model of the dual QCD ground state. The quantum part of VED (4.2) always contains imaginary part. Thus, the vacuum of the Abelian Higgs model without string contributions is unstable indeed. Whether the string contributions can cure this fundamental problem or not is beyond the scope of this letter and is left for consideration elsewhere. The vacuum instability of the Abelian Higgs model without string contributions will be recovered, of course, within A-scheme (2.16) as well. Nothing depends on how one introduces the scale dependence by choosing different scale parameters.

Comparing Eqs. (2.9) and (2.15), a prescription to obtain the relevant expression for the truly nonperturbative VED can be derived. Indeed, for this purpose in Eq. (2.9) the replacement  $q^2 F(q^2) + G(q^2) \rightarrow 1 + q^2 F^{NP}(q^2) + G^{NP}(q^2)$  should be done. Also the soft cutoff  $q_0^2$  on the upper limit should be introduced. Now it looks like the UV cutoff, but nevertheless let us underline once more that it separates the deep IR region from the perturbative one, which includes the IM region as well. It can not be arbitrary large as the UV cutoff is, by definition. As far as one chooses Ansatz for the full gluon propagator, the separation "NP versus PT" in Eq. (2.10) is exact because of the definition (2.11). The separation "soft versus hard" momenta is also exact because of the above-mentioned minimization procedure. Thus the proposed determination of the truly nonperturbative VED is uniquely defined. It is possible to minimize the effective potential at a fixed scale (2.17) with respect to the physically meaningful parameter. When it is zero, the perturbative phase only survives in all quantum models of the QCD ground state. Equivalently, we can minimize the auxiliary effective potential (2.22) as a function of the soft cutoff itself. As was underlined above, both methods lead to the same numerical value for the truly nonperturbative VED.

There is no general method to formulate in order to calculate the confining quark quantum contribution into the total VED since this contribution depends heavily on the particular solutions to the quark SD equation. If it is correctly calculated then it is of opposite sign to the nonperturbative gluon part and it is one order of magnitude less as well (see, for example our recent papers [19,23,28]). Concluding, let us note that the generalization of our method on different noncovariant gauges [26,30] is straightforward. Let us underline that our method is not a solution to the above-mentioned fundamental badly divergent problem of VED. However, it is a general one and can be applied to any nontrivial QCD quantum vacuum models in order to extract the finite part of the truly nonperturbative VED in a

self-consistent way. In particular, it can serve as a test of different axial gauge QCD quantum as well as classical vacuum models since our method provides an exact criterion for the separation "stable versus unstable vacua".

### ACKNOWLEDGMENTS

One of the authors (V.G.) is grateful to M. Polikarpov, M. Chernodub, A. Ivanov and especially to V.I. Zakharov for useful and critical discussions which led finally the authors to the formulation of a general method presented here. He also would like to thank H. Toki for many discussions on this subject during his stay at RCNP, Osaka University. This work was supported by the OTKA grants No: T016743 and T029440.

## REFERENCES

- [1] P. van Baal (Ed.), Confinement, Duality, and Nonperturbative Aspects of QCD, NATO ASI Series B: Physics, vol. 368;  
K-I. Aoki, O. Miymura, T. Suzuki (Eds.), Non-Perturbative QCD, Structure of the QCD Vacuum, Prog. Theor. Phys. Suppl. **131**, 1 (1998)
- [2] T. DeGrand, A. Hasenfratz, T.G. Kovacs, Nucl. Phys. **B505**, 417 (1997);  
D.A. Smith, M.J. Teper, Phys. Rev. D **58**, 014505 (1998)
- [3] V.N. Gribov, Eur. Phys. Jour. C **10**, 91 (1999)
- [4] J. Hosek, G. Ripka, Z. Phys. A **354**, 177 (1996); H.G. Dosch, Prog. Part. Nucl. Phys. **33**, 121 (1994); Yu.A. Simonov, Phys. Usp. **166**, 337 (1996)
- [5] T. Schafer, E.V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998)
- [6] J.M. Cornwall, R. Jackiw, E. Tomboulis, Phys. Rev. D **10**, 2428 (1974)
- [7] A. Barducci et al., Phys. Rev. D **38**, 238 (1988)
- [8] R.W. Haymaker, Riv. Nuovo Cim. **14**, 1-89 (1991);  
K. Higashijima, Prog. Theor. Phys. Suppl. **104**, 1-69 (1991)
- [9] W. Marciano, H. Pagels, Phys. Rep. C **36**, 137 (1978)
- [10] C.D. Roberts, A.G. Williams, Prog. Part. Nucl. Phys. **33**, 477 (1994);  
V. Gogohia, Gy. Kluge, M. Prisznyak, hep-ph/9509427
- [11] M. Baker, C. Lee, Phys. Rev. D **15**, 2201 (1977)
- [12] E.G. Eichten, F.L. Feinberg, Phys. Rev. D **10**, 3254 (1974)
- [13] J.E. Mandula, Phys. Rep. **315**, 273 (1999)
- [14] M.S. Chanowitz, S. Sharpe, Nucl. Phys. **B222**, 211 (1983)
- [15] R.J. Crewther, Phys. Rev. Lett. **28**, 1421 (1972);  
M.S. Chanowitz, J. Ellis, Phys. Rev. D **7**, 2490 (1973);  
J.C. Collins, A. Duncan, S.D. Joglecar, Phys. Rev. D **16**, 438 (1977)
- [16] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. **B147**, 385 (1979)
- [17] G.A. Christos, Phys. Rep. **116**, 251 (1984)
- [18] E. Witten, Nucl. Phys. **B156**, 269 (1979);  
G. Veneziano, Nucl. Phys. **B159**, 213 (1979)
- [19] V. Gogohia, H. Toki, Phys. Lett. B **466**, 305 (1999);  
V. Gogohia, H. Toki, Phys. Rev. D **61**, 036006 (2000), see also hep-ph/9908301
- [20] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. **B191**, 301 (1981)
- [21] I. Halperin, A. Zhitnitsky, Nucl. Phys. **B539**, 166 (1999)
- [22] G. Gabadadze, Phys. Rev. D **58**, 094015 (1998)
- [23] V. Gogohia, Gy. Kluge, H. Toki, T. Sakai, Phys. Lett. B **453**, 281 (1999)
- [24] M.N. Chernodub, M.I. Polikarpov, V.I. Zakharov, hep-ph/9903272
- [25] M. Baker, J.S. Ball, F. Zachariasen, Phys. Rev. D **37**, 1036 (1988)
- [26] A.I. Alekseev, B.A. Arbuzov, Phys. Atom. Nucl. **61**, 264 (1998);  
A.I. Alekseev, B.A. Arbuzov, Mod. Phys. Lett. A **13**, 1747 (1998)
- [27] P. Castorina, S.-Y. Pi, Phys. Rev. D **31**, 411 (1985)
- [28] V. Gogohia, H. Toki, T. Sakai, Gy. Kluge, Inter. Jour. Mod. Phys. A **15**, 45 (2000), see also hep-ph/9810516
- [29] N.K. Nielsen, P. Olesen, Nucl. Phys. **B144**, 376 (1978)
- [30] G. Leibbrandt, Noncovariant gauges, (World Scientific, Singapore, 1994)